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A COHERENT NONLINEAR THEORY OF AURORAL KILOMETRIC RADIATION.(U)

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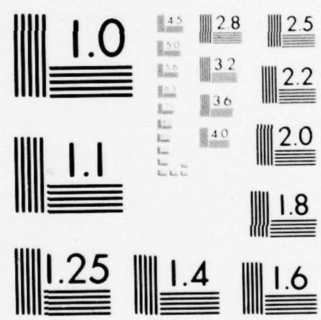

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A Coherent Nonlinear Theory of Auroral Kilometric Radiation

CROCKETT L. GRABBE

Science Applications, Inc.
McLean, Virginia 22102

P. J. PALMADESSO AND K. PAPADOPOULOS

Plasma Physics Division

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A theory of auroral kilometric radiation due to the nonlinear interaction between negative energy electromagnetic waves and coherent EIC waves is developed. The theory predicts that such radiation produced must have X-mode polarization and must have a frequency just slightly below the Doppler-shifted beam cyclotron frequencies. The basic requirement for free space accessibility is the presence of high-energy beams and a density cavity of the type observed by Isis I measurements. It is predicted that a harmonic band structure in the kilometric radiation may be observable.

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Contents

A COHERENT NONLINEAR THEORY OF AURORAL KILOMETRIC RADIATION

I. INTRODUCTION

There have been several observational studies made in recent years of high intensity kilometric radiation of terrestrial origin. Measurements have been made by satellite from the source region, in the auroral zone at $R \sim 2\text{-}3\ R_E$, all of the way out to $R \sim 25\text{-}30\ R_E$, far away from the source region.

A number of interesting features of the auroral kilometric radiation (AKR) have been deduced as a result of these satellite measurements. The first measurements were made on Ogo I [*Dunkel, et al.*, 1970]. More detailed measurements were made on Imp 6 and Imp 8, which were analyzed by *Gurnett, et al.* [1974]. Gurnett found the spectrum to lie in 50 kHz < f < 500 kHz, with peak intensity at $f \sim 200$ kHz. The peak intensity power emission was estimated to be about 10^9 W. He noted that the radiation appears to originate at low altitudes ($R \sim 2\text{-}3\ R_E$) in the auroral region, and to be closely associated with the occurrence of discrete auroral arcs, which are believed to be generated by intense inverted V electron precipitation bands. When arcs do not occur, only a small band of diffuse aurora is present, and the radiation disappears. The radiated power was estimated to be close to 1 percent of the maximum energy dissipated by the auroral charged particle precipitation.

Recent observations in the source region from Isis I reveal and establish further properties of the AKR, which were not well-established from the far-field measurements [*Benson and Calvert*, 1979]. Those

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observations showed that the radiation was generated in the X-mode just above the local cutoff frequency, and propagated almost perpendicular to the background magnetic field. It was found to be generated within density-depleted regions with peak density such that $\omega_{pe} < 0.2\ \omega_{ce}$. (These are probably caused by the large potential drop in those regions associated with the discrete auroral arcs.)

A number of theories have been proposed in the literature to explain the occurrence of the high-intensity radiation. *Oya* [1974] proposed a mechanism for the well-known Jovian decametric radiation (thought to have a similar origin as that of the terrestrial kilometric radiation) which involved the creation of electromagnetic radiation by a mode conversion process. This mechanism was extended to kilometric radiation by *Benson* [1975]. In this model, excited plasma waves were converted into the X-mode at the upper hybrid frequency. This mode was then converted to the O-mode. The overall efficiency of this mechanism is estimated to be a few percent.

There are two major drawbacks with this theory. First, the polarization is in disagreement with recent polarization measurements, including both far-field measurements [*Gurnett and Green*, 1978] and measurements at the source [*Benson and Calvert*, 1979]. Second, the overall efficiency, which is the product of the efficiencies of two inefficient conversion mechanisms is too small. This does not even take into account the conversion efficiency of precipitating electrons into electrostatic plasma waves in the first place. Thus it is highly unlikely that this indirect mechanism could produce final output

radiation which is on the order of 1 percent of precipitating electron (or electron beam) energy.

Melrose [1976] proposed a theory for the Jovian decametric and terrestrial kilometric radiation which involves directly amplified gyro-emissions. In this mechanism, radiation is emitted at the electron cyclotron frequency and its harmonics, which have been Doppler Shifted by electron streaming. This radiation is then amplified due to assumed anisotropic velocity distribution functions. It is also assumed that the electron streaming velocity is large enough to allow the radiation to escape to free space.

The frequency for the radiation required in Melrose's theory will turn out to be the most favorable frequency for radiation in the theory proposed in the present paper. However, to get growth rates required to produce kilometric radiation at the observed level by anisotropic velocity space instabilities would appear to require very large anisotropies in velocity space. There is at present no experimental evidence for such large anisotropies.

Wu and Lee [1979] have recently proposed a mechanism which involves a velocity-space instability, namely a loss-cone type instability. In their model, precipitating electrons stream down the (converging) magnetic field lines toward Earth until they encounter an effective magnetic mirror in a region of higher magnetic field strength. They then undergo reflection and travel up diverging magnetic field lines. Some of the upgoing particles are lost through the loss cone, and the **resulting** velocity distributions lead to a loss cone instability.

Wu and Lee incorporate into their model a long density-depleted cavity in which the unstable wave grows, as found by Benson and Calvert on Isis I. This cavity was not considered in any of the previous papers, but will be assumed in our theory. The most probable explanation is that the cavity is formed by large potential drops known to be present in the auroral region, which also tend to drive large electron (and even ion) beams. For example, if we assume the background density is close to thermodynamic equilibrium, then the density profile is determined by the Boltzmann form

$$n_e(x) = n_o \exp [- e\phi(x)/\kappa T_e]$$

where $\phi(x)$ is the local potential and T_e the electron temperature. Thus the regions of high potential have lower density.

One drawback of the Wu-Lee theory is that to get the large growth lengths required, loss-cone angles of $\theta > 45$-$60°$ appear to be required, as for example, the effective angles in the distribution functions assumed by Wu and Lee. The observed loss cone angles in the region of origin are more like $\theta \sim 15$-$20°$.

Another recent theory for kilometric radiation was proposed by *Roux and Pellat* [1979]. Their mechanism involved the nonlinear beating of electrostatic waves near the upper hybrid to produce an electromagnetic wave. Their theory predicts that the O-mode should dominate at $\omega = \omega_{uh}$ (at the upper hybrid frequency) for $\omega_{pe} > \omega_{ce}$, and that the X-mode should dominate at $\omega = 2\,\omega_{uh}$ for $\omega_{pe} < \omega_{ce}$.

Roux and Pellat argue that a coherent nonlinear theory, such as their proposal or the present proposal, should be more efficient in general than an incoherent linear mechanism or even a coherent linear mechanism. However, their particular nonlinear mechanism, which requires the creation of a high phase velocity transverse wave from two low phase velocity longitudinal waves, would be expected to be rather inefficient. Also, their prediction of radiation in the X-mode at $\omega = 2\omega_{uh}$ for the density depleted case observed is in apparent disagreement with the observation that the frequency at the origin is just above the right-hand cutoff frequency. Roux and Pellat also mention the possibility of the nonlinear interaction of an upper hybrid wave and a lower hybrid wave to produce radiation at $\omega = \omega_{uh} + \omega_{\ell h}$. However, since $\omega_{pe} << \omega_{ce}$ in the density cavity then this would give $\omega \cong 1.02\,\omega_{ce}$, and the resulting radiation may be below the right hand cutoff. Also, the efficiencies for this case would appear to be less than the case for two upper hybrid waves, but it has not yet been analyzed.

Finally, a theory proposed by *Palmadesso, et al.* [1975], assumed that electromagnetic "noise" gets amplified through its interaction with ion turbulence. The nonlinear beat wave of these two waves could be amplified by beams, and thus the electromagnetic wave would be parametrically amplified. We will employ a similar mechanism in the theory presented in the present paper.

In the Palmadesso theory, both the RX and LO modes were assumed to be amplified by the above mechanism. However, for the beam to significantly amplify the beat wave, and hence the electromagnetic wave, the

beat wave was assumed to be almost a natural mode of the plasma, and required to have a slow phase velocity ($\omega/k \ll c$). This required generation at frequencies below the upper hybrid. Thus the RX mode would encounter a cutoff at frequencies between the upper hybrid and the right-hand cutoff and not escape to free space. Thus, only the LO mode would be observed, contrary to recent polarization measurements. Also, the ion waves were taken to be incoherent (turbulent), and assumed to be isotropic for convenience of calculation. Recent measurements of EIC waves, e.g., *Temerin, et al.* [1978], show strong electrostatic ion cyclotron (EIC) wave fluctuations which are coherent and primarily perpendicular to the background magnetic field. The coherence of the waves was unanticipated and suggests a modified application of the mechanism in *Palmadesso, et al.*, can be employed.

We will construct the present theory with the same principle underlying mechanism. However, we will construct a new theoretical framework so the formulation of the theory will be different. We will assume coherent electrostatic density fluctuations (due to low-frequency EIC waves) perpendicular to the magnetic field. We will concentrate on the case that there is a resonant interaction between electromagnetic wave and the coherent density fluctuations, a case not considered by *Palmadesso, et al.* It will be shown that the X-mode is preferentially amplified over the O-mode in this model, and that the X mode can grow rapidly in a narrow frequency band that is accessible to free space. The basic requirement for this last condition to exist will be shown to be that a density cavity of precisely the density observed is present at the source.

In Sec. II we will introduce the basic model for the AKR, derive the wave equation and associated dispersion relation, and discuss solutions. In Sec. III we will analyze the wave equation to find the instability criterion and the growth rate for the unstable modes. In Sec. IV we discuss in more detail the physics of the wave amplification. Results are summarized in Sec. V.

II. MODEL AND GEOMETRY

We assume that initially we have electromagnetic radiation propagating at the noise level in the auroral zone, which has been generated by one of a variety of possible mechanisms: Cerenkov, electron cyclotron, bremstrahlung, etc. We also assume that there are coherent density fluctuations perpendicular to the $\vec{B}$-field due to EIC waves, as observed by **Temerin and coworkers.** The background magnetic field $\vec{B}_o = B_o\hat{z}$ is taken along the z direction. The electromagnetic wave is assumed to propagate almost perpendicular to the magnetic field, with propagation vector $\vec{k} = (k_x, 0, k_z)$, where $k_z^2 << k_x^2$, as observed by *Benson and Calvert* [1979]. (See Fig. 1.)

There are two independent modes of the electromagnetic wave, with $\vec{E}$-field vectors:

$$\vec{E} = \left[E_x(x)\hat{x} + E_y(x)\hat{y}\right] \; e^{i(k_z z - \omega t)} \qquad \text{X-mode} \qquad (1a)$$

$$\vec{E} = \left[E_x(x)\hat{x} + E_z(x)\hat{z}\right] \; e^{i(k_z z - \omega t)} \qquad \text{O-mode} \qquad (1b)$$

We are dealing with high-frequency waves this model so that $\omega \gtrsim \omega_{ce}$, ω_{pe}. The waves are primarily transverse waves, so $E_x << E_y, E_z$.

The EIC waves have associated density fluctuations of both ions and electrons; the electrons are assumed to follow the ions in the fluctuations so that the resulting total plasma fluctuation is quasi-neutral. Then the plasma density is of the form

$$n(x,t) = n_o + n_1(x,t) \qquad (2a)$$

$$n_1(x,t) = \delta n \cos(k_i x - \omega_i t) \qquad (2b)$$

where n_o is the background density, n_1 is the fluctuating density, and ω_i and k_i are the frequency and the wavenumber of the EIC wave. Since $\omega_i \sim \omega_{ci} << \omega_{ce} \sim \omega$, the electromagnetic wave sees the density fluctuation as stationary. Thus we can ignore the $\omega_i t$ term in n_1 in our electromagnetic wave equation:

$$n_1 \approx \delta n \cos k_i x \tag{2c}$$

The wave equation for the electromagnetic wave is

$$\nabla \times (\nabla \times \vec{E}) + \left(\frac{\omega}{c}\right)^2 \overleftrightarrow{K} \cdot \vec{E} = 0 \tag{3}$$

where $\overleftrightarrow{K}$ is the plasma dielectric tensor, and is of the form

$$\overleftrightarrow{K} = \begin{bmatrix} K_\perp & -iK_H & 0 \\ iK_H & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{bmatrix} \tag{4}$$

where $K_\perp$ and $K_\parallel$ are the dielectric constants perpendicular to and along the magnetic field, respectively, and K_H is the Hall term. We will divide the components of $\overleftrightarrow{K}$ according to

$$K_{ij} = K_{ijo} + \delta K_{ij} \tag{5}$$

where K_{ijo} are the tensor components for the case of $n(x,t) = n_o$, and δK_{ij} are the perturbations introduced in by the density fluctuation n_1. Including the effect of a beam of velocity $\vec{v}_b = v_b \hat{z}$ along the $\vec{B}$-field, the dielectric tensor components for the nonfluctuating

background plasma for the case of a cold plasma and a warm beam are:

$$K_{\perp o} = 1 - \frac{\omega_{peo}^2}{\omega^2-\omega_{ce}^2} - \frac{\omega_b^2}{(\omega-k_z v_b+i\nu)^2-\omega_{ce}^2} \tag{6a}$$

$$K_{Ho} = \frac{\omega_{ce}}{\omega}\left[\frac{\omega_{peo}^2}{\omega^2-\omega_{ce}^2} + \frac{\omega_b^2}{(\omega-k_z v_b+i\nu)^2-\omega_{ce}^2}\right] \tag{6b}$$

$$K_{\parallel o} = 1 - \frac{\omega_{peo}^2}{\omega^2} - \frac{\omega_b^2}{(\omega-k_z v_b+i\nu)^2} \tag{6c}$$

with ω_{peo} the plasma frequency for plasma density n_o. We have neglected the low-frequency ion terms, since they are not of interest for the high-frequency electromagnetic wave. Here ω_b is the beam-plasma frequency, and ν enters because of the finite beam width themal effects for an assumed Lorentzian beam velocity profile. In terms of the beam spread Δv in velocity space,

$$\nu = k_z \Delta v \tag{7}$$

The use of a Lorentzian beam profile for thermal effects is valid provided $v_b >> \bar{v}_e \sim \Delta v$ where $\bar{v}_e$ is the average electron thermal velocity of the beam.

From the form of $\vec{E}$ for the X and O mode given by Eq. (1) we may obtain a reduced wave equation for each mode through Eq. (3).

$$\frac{\partial^2 E_y}{\partial x^2} + (\alpha_1 - \epsilon\alpha_2 \cos k_i x)\, E_y = 0 \qquad \text{X-mode} \tag{8a}$$

$$\frac{\partial^2 E_z}{\partial x^2} - (\beta_1 + \epsilon\beta_2 \cos k_i x)\, E_z = 0 \qquad \text{O-mode} \tag{8b}$$

where

$$\varepsilon = \frac{\delta n}{n_o} \tag{9a}$$

$$\alpha_1 \cong \left(\frac{\omega}{c}\right)^2 \left[K_{\perp o} - \frac{K_{Ho}^2}{(K_{\perp o} - n_z^2)} - n_z^2\right] \tag{9b}$$

$$\alpha_2 \cong \frac{\omega^2 \omega_{peo}^2}{c^2(\omega^2 - \omega_{ce}^2)} \left[1 + \frac{2K_{Ho}^2}{(K_{\perp o} - n_z^2)^2} + \left(\frac{\omega_{ce}}{\omega}\right) \frac{K_{Ho}}{(K_{\perp o} - n_z^2)}\right] \tag{9c}$$

$$\beta_1 \cong \frac{(K_{\perp o} - n_z^2) K_{\parallel o}}{K_{\perp c}} \tag{9d}$$

$$\beta_2 \cong K_{\parallel o} \left(\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}\right) n_z^2 - \frac{\omega_{pe}^2}{\omega^2} \left(\frac{K_{\perp o} - n_z^2}{K_{\perp o}}\right) \tag{9e}$$

The form of Eq. (8) is obtained under the assumptions:

$$|\omega^2 - \omega_{uh}^2| >> \varepsilon\, \omega_{pe}^2 \tag{10a}$$

$$\varepsilon^2 << 1 \tag{10b}$$

Since measured density fluctuations give $\varepsilon < 0.5$, the $\varepsilon^2 << 1$ assumption appears to be quite good. Also, Eq. (10a) is quite valid except for a very narrow range around the upper hybrid frequency:

$$1.01\, \omega_{uh} \gtrsim \omega \gtrsim 0.99\, \omega_{uh} \tag{11}$$

The latter frequency range will not be of interest to us because it lies below the right hand cutoff, and involves a wave inaccessible to free space.

Thus the wave equations for both the X and the O mode reduce to Mathieu's equation for the model being used here. From this equation

we may find stability conditions and growth rates which arise from resonant spatial interaction between the electromagnetic wave and spatial density fluctuations, as well as the modification of the electromagnetic waves by the density fluctuations.

For purposes of illustration of the solution, we will solve the wave equation for the X-mode. The O-mode solution is quite similar. In obtaining Eq. (8a), we used the relationship between E_x and E_y from Eq. (3). From that, the appropriate form of the solution for $\vec{E}$ of the X-mode is

$$\vec{E} = A^{\pm} \left(\hat{y} - \frac{K_{Ho}}{(K_{1o} - n_z^2)} \hat{x} \right) e^{i(k_z z - \omega t)}$$

$$\times \sum_n^{\pm} a_n^{\pm} e^{\pm i(q + nk_i)x} \tag{12}$$

where $a_o^{\pm} = 1$, and for no density fluctuations

$$a_n^{\pm} (\varepsilon = 0) = 0 \quad (n \neq 0) \tag{13}$$

By substituting this form into our wave equation, Eq. (3), we find the dispersion relation to order ε^2

$$q^6 - q^4 (2k_i^2 + 3\alpha_1) - q^2 \left(\frac{\varepsilon^2 \alpha_2^2}{2} - k_i^4 - 3\alpha_1^2 \right)$$

$$- \left[2\alpha_1 (\alpha_1 - k_i)^2 + \varepsilon^2 \alpha_2^2 (k_i^2 - \alpha_1) \right] + O(\varepsilon^4) = 0 \tag{14}$$

Also, solution of the coefficients to order ε yields

$$a_1^{\pm} = \frac{-\varepsilon\alpha_2}{2[\alpha_1 + (q - k_i)^2]} \tag{15a}$$

$$a_{-1}^{\pm} = \frac{-\varepsilon\alpha_2}{2[\alpha_1 + (q-k_i)^2]} \tag{15b}$$

These equations show the interaction between the electromagnetic wave and the first harmonic of the density fluctuation. If we included in higher orders of ε, we would get the higher harmonics. In general, for the inclusion of harmonics up to n, we get a dispersion relation of order 2n+1 in q^2, and coefficients $a_n^{\pm} \sim \varepsilon^n$. It should be noted that in the limit $\varepsilon \to 0$, the dispersion relation in Eq. (14) has roots

$$q_{1,2} = \pm \sqrt{\alpha_1} \tag{16a}$$

$$q_{3,4} = \pm \left(\sqrt{\alpha_1} - k_i\right) \tag{16b}$$

$$q_{5,6} = \pm \left(\sqrt{\alpha_1} + k_i\right) \tag{16c}$$

The first two roots are just the fundamental mode, and the last four are simple first harmonics. However, for $\varepsilon \to 0$, the amplitudes $a_{\pm 1}^{\pm}$ of these harmonics are zero.

III. STABILITY CRITERION AND GROWTH RATES

When the electromagnetic wave is propagating (i.e., for $\alpha_1 > 0$), then the q's in Eq. (16) for the unmodulated wave are real. However, the presence of density fluctuations (so $\varepsilon \neq 0$) causes the roots for q in Eq. (14) to be complex for certain conditions, so that the propagation wave has growing or decaying solutions. The general requirement for such an instability to exist is

$$\sqrt{\alpha_1} \simeq pk_i \qquad \text{(X-mode)}, \tag{17a}$$

$$\sqrt{\beta_1} \simeq pk_i \qquad \text{(O-mode)} \tag{17b}$$

where p is a positive integer. The larger ε is, the less strictly this criterion has to be satisfied. A stability diagram is shown in Fig. 2. Basically Eq. (17) is just the condition for resonant interaction between the electromagnetic wave and a spatial harmonic of the density fluctuation. As one would expect, the p = 1 model is the fastest growing.

The conditions for instability of the X-mode may be rewritten

$$p^2 \left(\frac{ck_i}{\omega}\right)^2 + n_z^2 = K_\perp + \frac{K_H^2}{(K_\perp - n_z^2)} \tag{18}$$

Generally, $ck_i/\omega \gtrsim 1$ since the ion wave wavelength tends to be shorter than the free-space electromagnetic wavelength (but not of the electromagnetic wavelength in the plàsma medium). Thus for there to be a solution of the X-mode stability criterion, we must have

$$\frac{\omega_b^2}{\omega_{ce}^2 - (\omega - k_z v_b)^2} \gtrsim \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \tag{19}$$

There are two frequency regimes where this is satisfied:

(i) $\omega \gtrsim \omega_{ce}$. This frequency regime has waves which are not accessible to free space, because of the cutoff layer between the upper hybrid and right-hand cutoff. Also, there may be electron cyclotron damping which would partially offset any growth of the wave. Thus this regime is unlikely to produce high-intensity radiation which is observable in free space.

(ii) $\omega \gtrsim \omega_{ce} + k_z v_b$. This is the Doppler-shifted beam-cyclotron resonance. There are waves in this regime which are accessible to free space provided

$$\omega_{ce} + k_z v_b > \omega_R = \frac{\omega_{ce}}{2} + \left(\omega_{pe}^2 + \frac{\omega_{ce}^2}{4}\right)^{1/2} \tag{20}$$

Expanding the right hand side for $\omega_{pe}^2 << \omega_{ce}^2$ this becomes

$$\omega_{pe}^2 < k_z v_b \omega_{ce} \tag{21}$$

For example, for $k_z \sim 0.3k$ and $v_b \sim c/4$, we find

$$\omega_{pe} < 0.3\ \omega \tag{22}$$

This is to be compared to $\omega_{pe} < 0.2\ \omega$ in the source of AKR. Thus the density depletion observed by Isis I at the source is of just the amount predicted by theory to be necessary for radiation. When the condition in Eq. (22) is satisfied, we get radiation in the narrow band

$$\omega_{ce} + \omega_{pe}^2/\omega_{ce} < \omega < \omega_{ce} + k_z v_b \tag{23}$$

just above the right hand cutoff. This frequency regime is then the most probable regime of the kilometric radiation. It should be noted that there is a range of values of the plasma and cyclotron frequencies at the source of AKR in the aurora, and the frequencies which lie in the narrow band given by Eq. (23) are different for each sourcepoint. Thus the total radiation received has the broad spectrum between 50 and 500 kHz observed. Also, Eq. (21) shows that radiation will not be observed in regions where the beams are not of sufficiently high energy, or where the plasma density is too high (such as in regions $R < 2\ R_E$, or outside of the density depleted region), or where the magnetic field is too low (such as in regions at $R > 3\ R_E$).

Having established the frequency regime for the radiation, it must also be pointed out Eq. (19) also gives a minimum required beam density n_b, namely

$$\left(\frac{n_b}{n_o}\right) = \left(\frac{\omega_b}{\omega_{pe}}\right)^2 > \frac{\nu^2 + \omega_{ce}^2 - (\omega - k_z v_b)^2}{\omega^2 - \omega_{ce}^2} \qquad (24)$$

where we have included in the finite beam width and damping term ν. On resonance, this reduces to

$$\left(\frac{n_b}{n_o}\right)_{min} > \left(\frac{\nu^2}{\omega^2 - \omega_{ce}^2}\right) = \left(\frac{k_z (\Delta v)^2}{2\ \omega_{ce} v_b}\right) \qquad (25)$$

which, for $\nu = k_z \Delta v \sim 0.3\ k_z v_b$, $k_z \sim 0.3\ k$, $v_b \sim 0.1\ c$ gives

$$n_b > 1.5 \times 10^{-3}\ n_o \qquad (26)$$

Recall n_o is the reduced plasma density due to density depletion, so in this case instability would be possible for $n_b > 6 \times 10^{-5}\, n_p$, where n_p is the background plasma density outside the density cavity.

For the O-mode we may make the same analysis as we did for the X-mode. For instability for this mode we require

$$p^2 \left(\frac{ck_i}{\omega}\right)^2 + n_z^2 = \left(\frac{K_{\perp o} - n_z^2}{K_{\perp o}}\right)\left(1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_b^2}{(\omega - k_z v_b)^2}\right) \tag{27}$$

Now the left hand side is always greater than one, whereas the right hand side is always less than one for propagation waves. Thus we have no unstable solution of this equation and high-intensity radiation is not possible. It should be recalled that Eq. (27) breaks down in a very narrow band near the upper hybrid. Thus the presence of a $k_z \neq 0$ makes it conceivable that there might be an instability and some radiation in the O-mode near $\omega = \omega_{uh}$. However, even if such an instability can exist, the band is very narrow, and the wave equation is not nearly as unstable as the Mathieu equation. Thus, in any case, the X-mode radiation would definitely dominate, and we will concentrate on that mode from now on.

We may now calculate the growth rate for our X-mode radiation. For example, the dispersion relation for the wave equation including the first harmonic interactions, Eq. (14), will yield an instability provided the equation has a complex (non-real) solution. The spatial growth rate for the first harmonic instability which occurs when $\sqrt{\alpha_1} \approx k_i$ ($p = 1$ mode) is for $\varepsilon \ll 1$

$$\kappa_1 \cong \frac{\sqrt{\alpha_1}}{4} \left[\left(\frac{\alpha_2}{\alpha_1}\right)^2 \frac{\varepsilon^2}{4} - \left(\frac{\sqrt{\alpha_1}}{k_i} - 1\right)^2 \right]^{1/2} \tag{28}$$

Note that as ε increases in size, both the maximum growth rate and the size of the unstable region increase. Similarly, for the p = 2 mode we have a growth rate

$$\kappa_2 \cong \frac{\sqrt{\alpha_1}}{4} \left[\left(\frac{\alpha_2}{\alpha_1}\right)^2 \frac{\varepsilon^2}{4} - \left(\frac{\sqrt{\alpha_1}}{k_i} - 2\right)^2 \right] \tag{29}$$

Plots of typical curves for the growth length $L = \kappa^{-1}$ for these two modes are shown in Figs. 3-6. As p increases, the maximum growth rate and the frequency width of the instability both decrease. Thus we are most interested in the two lowest modes. Recent measurements indicate a harmonic band structure, and our theory just predicts that these harmonic bands are the various p mode harmonics.

The graphs show that when the conditions are right, the wave has a growth length of $L \sim 5$-15 km for the p = 1 mode. Similarly, the growth length for p = 2 can be $L \sim 10$-30 km. Assuming the kilometric radiation intensity to be $\sim 10^5$-10^6 above the noise level of electromagnetic radiation at the source, a total path length $L_{tot} \sim 70$-150 km is required to produce the radiation observed.

A schematic diagram of a possible ray path in the auroral density depleted cavity is shown in Fig. 7. The cavity width may be ~ 50-100 km, which is a typical high-energy beam region width. Radiation may then undergo multiple reflections off of the density cavity boundaries, where the density is higher, if the right hand cutoff layer is encountered there, and amplify on each pass. The cavity length may typically

be $\sim$ 1000-2000 km, enough to allow the wave to propagate to a region where the background density is such that at the cavity boundary is then accessible to free space. Note that wave propagating in directions toward higher magnetic fields and higher background density will eventually reach the right hand cutoff layer and reflect back toward lower field and density regions. Thus this radiation may also reach a region of escape to free space. In any case, the path length available is quite adequate to produce the growth required.

IV. PHYSICAL MECHANISM

Now that we have found the unstable solutions to the wave equation and shown that they are adequate to explain the kilometric radiation observed, it is informative to look at the physics of the amplification in more detail.

As shown in Section III, there were two frequency regimes in which growing waves could exist: $\omega \gtrsim \omega_{ce}$ and $\omega \gtrsim \omega_{ce} + k_z v_b$. The former was discounted because of inaccessibility and possible cyclotron damping problems. Let us consider the energy density of the wave in the two frequency ranges as seen by the electrons in their rotating frame:

$$U = \frac{\varepsilon_o}{2} \vec{E}^* \cdot \frac{\partial}{\partial \omega'}(\omega' \vec{\vec{K}}'_o) \cdot \vec{E} = 0 \tag{30}$$

where

$$\vec{\vec{K}}'_o = 1 - \frac{\omega_{pe}^2}{\omega'^2} - \frac{\omega_b^2}{(\omega' - k_z v_b)^2} \tag{31}$$

and $\omega' = \omega - \omega_{ce}$ = Doppler shifted frequency seen by rotating electron.

$$U = \frac{\varepsilon_o}{2} \left[\left(1 + \frac{\omega_{pe}^2}{\omega'^2} + \frac{\omega_b^2 (\omega' + k_z v_b)}{(\omega' - k_z v_b)^3} \right) \left(|E_x|^2 + |E_y|^2 \right) \right] \tag{32}$$

Now U is negative when $\omega' \gtrsim k_z v_b$, i.e., near the Doppler shifted cyclotron resonance $\omega \gtrsim \omega_{ce} + k_z v_b$ the X-mode is a negative energy mode. However, for $\omega \gtrsim \omega_{ce}$, it is very difficult to get a negative energy mode. Thus the former mode can absorb energy from the beam, whereas the latter cannot, because of the relation of the phase velocity in the electron frame ω'/k_z to the beam velocity v_b.

When the frequency range $\omega \gtrsim \omega_{ce}$ is not a negative energy mode, any energy it absorbs must come from the EIC density fluctuations via resonant interaction. [See Fig. 8(a).] Ordinarily, energy flow from the electromagnetic wave to the lower energy ion waves would be thermodynamically favored, and the X-mode is more likely to be absorbed in this frequency range. However, in the frequency range $\omega \gtrsim \omega_{ce} + k_z v_b$ energy flow from the beam into the negative energy X-mode via coherent interaction with the ion mode is thermodynamically favored. [See Fig. 8(b).]

In Fig. 8(b) it is seen that the beam interacts with and effectively amplifies the nonlinear beat wave between the EIC mode and the X-mode. This mode is a driven or virtual mode, also called a quasi-mode since it is not necessarily a natural mode of the system. We may learn more about this nonlinearly driven mode by looking at its phase, which is the phase difference between the modes producing it. For interaction with the pth spatial harmonic

$$\phi(x,t) = (k_x - pk_i)x - (\omega - \omega_i)t + k_z z \tag{33}$$

Now the EIC wave frequency $\omega_i << \omega$, and for instability we require $k_x = \sqrt{\alpha_1} \cong pk_i$. Thus

$$\phi(x,t) \cong k_z z - \omega t \tag{34}$$

which is just the phase of the right-hand circularly polarized (R) mode travelling almost along the magnetic field. This mode has an unstable resonance at the Doppler-shifter beam cyclotron resonance and is thus

amplified by the beam. The amplification of the virtual mode then amplifies the parent waves which are driving it. It should be noted that electromagnetic noise is not even necessary initially. The beam may drive an R-mode, which interacts with EIC waves to produce X-mode waves. (See Fig. 9.)

V. SUMMARY AND CONCLUSIONS

Nonlinear amplification of electromagnetic wave through coherent interaction with EIC waves and electron beams for the model used appears to be quite adequate to produce the power levels of kilometric radiation observed. All aspects of the model and results appear to be in good agreement with current observations. The radiation is predicted to lie in a narrow frequency band just above the local right-hand cutoff, as observed at the source. The basic conditions for this to occur in our theory is that we have a high-energy beam, and a density-depleted cavity of precisely the type observed. For large growth due to interaction with density fluctuations, the radiation must propagate almost perpendicular to the background magnetic field and have X-mode polarization, both characteristics just as observed by Isis I source measurements. The existence of the various p-mode instabilities indicates a harmonic band structure consisting of the lowest p-modes might be observable, in agreement with recent measurements [*Shawhan,* 1979].

For the $p = 1$ mode, growth lengths of $L \sim 100$ km seem to be adequate to bring the kilometric radiation out of the noise to the power levels observed. Imperfections in the source region may raise this somewhat. Higher order modes may take several hundred kilometers. Such path lengths appear to be easily obtained.

The locally generated frequency should decrease as the radius R increases. The model implies that the reason the radiation is restricted to $R \sim 2$-$3\ R_E$ is that for $R < 2\ R_E$ the plasma density is too high and for $R > 3\ R_E$ the magnetic field is too low, to put the Doppler-shifted

beam cyclotron frequency above cutoff. Only for $R \sim 2\text{-}3\ R_E$ can radiation be generated above cutoff.

It should be noted that there are possible future applications of the proposed mechanism for generation of radiation to laboratory sources of high-intensity radiation, and possibly even plasma-based masers or lasers. This is related to the *Kaw, Lin and Dawson's* [1973] proposal for a plasma laser. A future paper on that area is planned.

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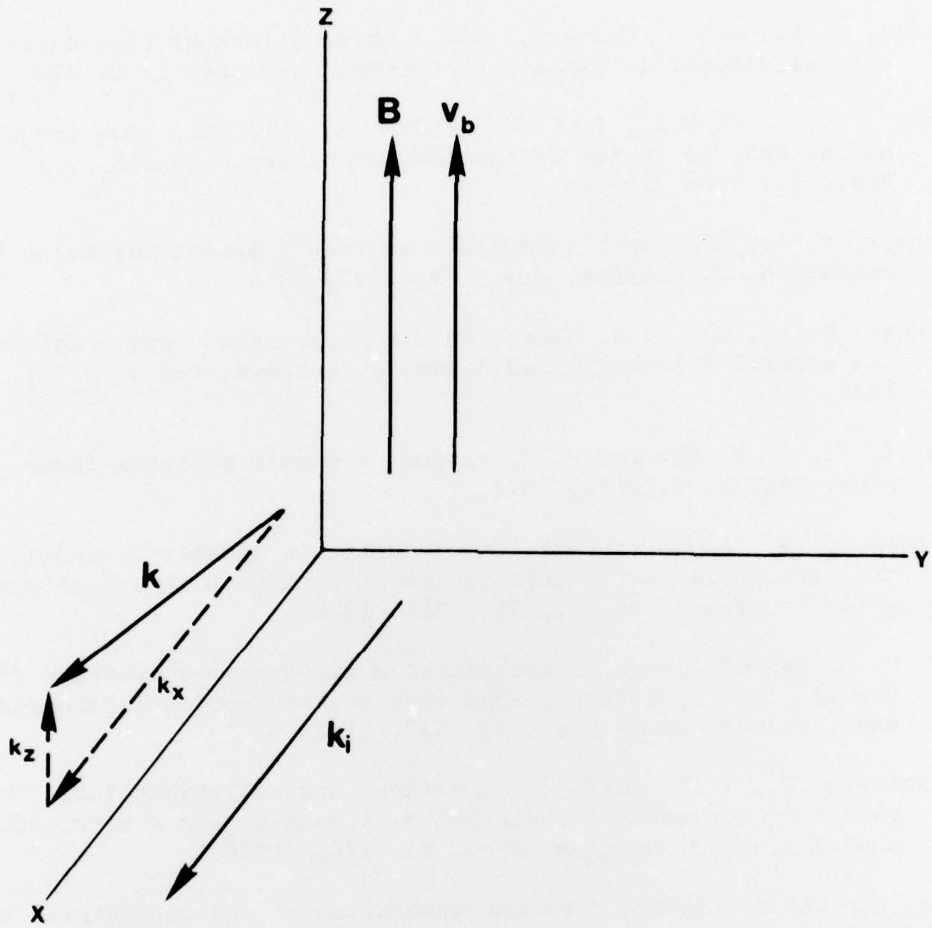

Fig. 1. Geometry of model. $\vec{k}$ is the propagation vector of the electromagnetic wave, and $\vec{k}_i$ the propagation vector for the EIC waves. The latter produce density fluctuations along x.

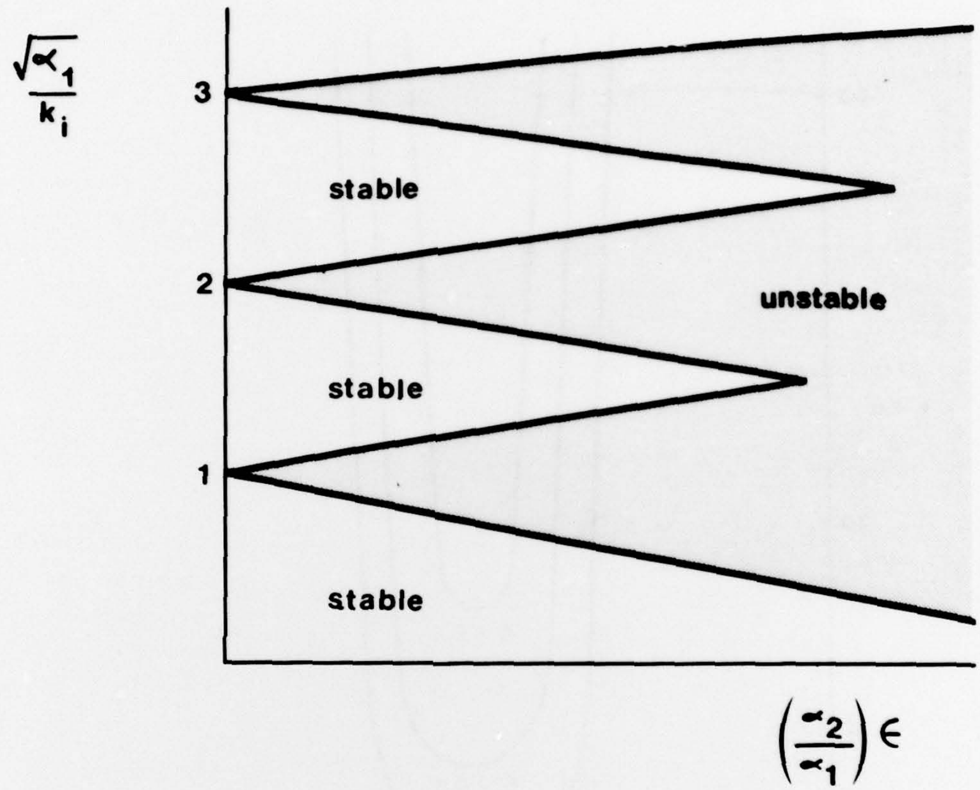

Fig. 2. Stability diagram for the wave equation of the X-mode. Instability occurs whenever $\sqrt{\alpha_1} \cong pk_i$, where p is an integer. As the relative density fluctuation amplitude ε increases, the less strictly this criterion must be satisfied.

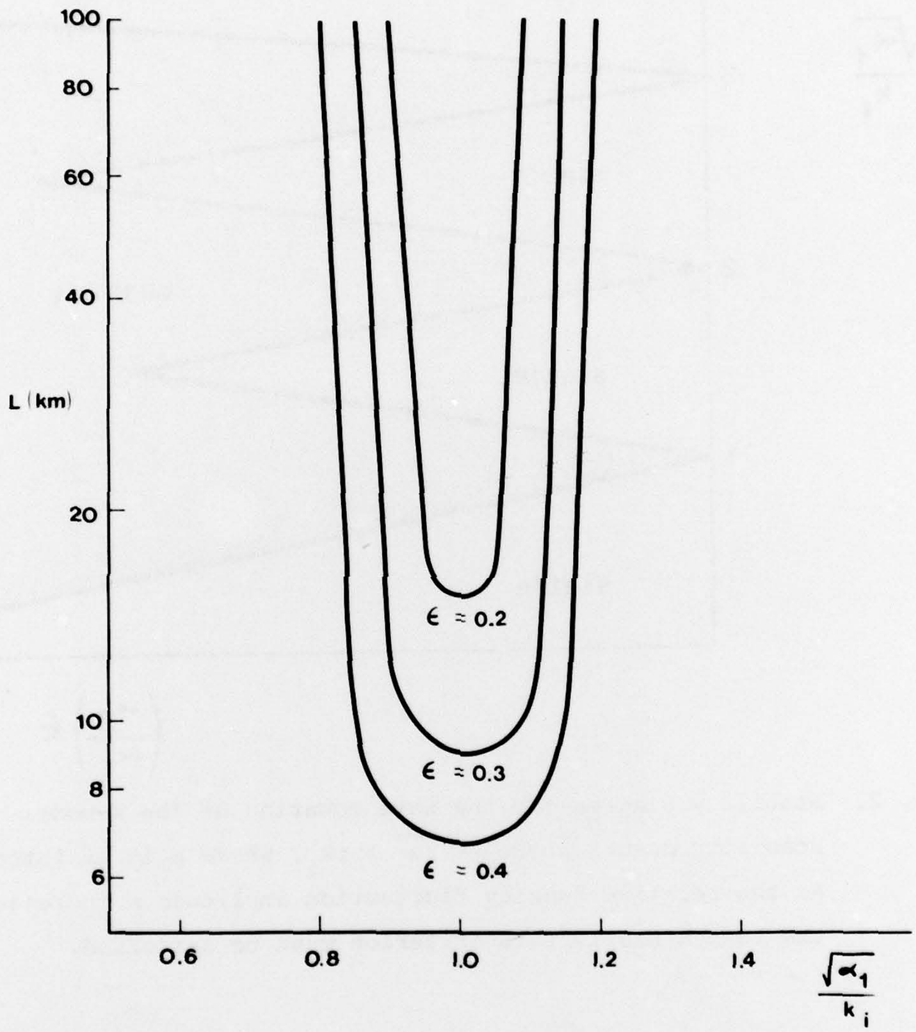

Fig. 3. Growth length L for the $p \approx 1$ mode for various density fluctuation ratios ε, and frequency $\omega \cong \omega_{ce} + k_z v_b$. The plasma parameters we have chosen are $f \cong 178$ kHz, $\omega_{pe} \cong 0.3\ \omega_{ce}$, $k_z \cong 0.3\ k$, $v_b \cong 0.2\ c$, $\Delta v \cong 0.3\ v_b$ and $n_b = 6 \times 10^{-3}\ n_o$.

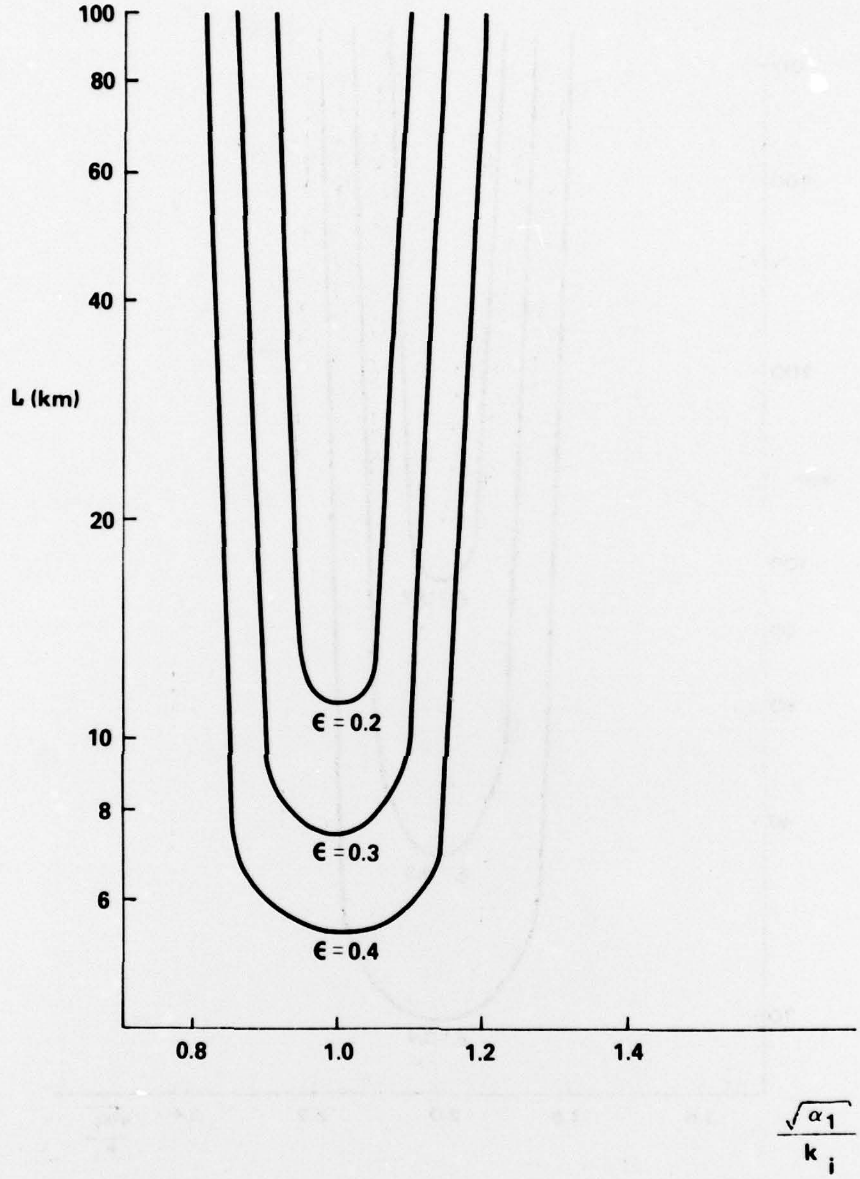

Fig. 4. Growth length for p = 1 mode for the following set of plasma parameters: $\omega \cong \omega_{ce} + k_z v_b$, $f \cong 178$ kHz, $\omega_{pe} \cong 0.3\ \omega_{ce}$, $k_z \cong 0.3\ k$, $v_b \cong 0.3\ c$, $\Delta v \cong 0.1\ v_b$ and $n_b \cong 10^{-2}\ n_p$.

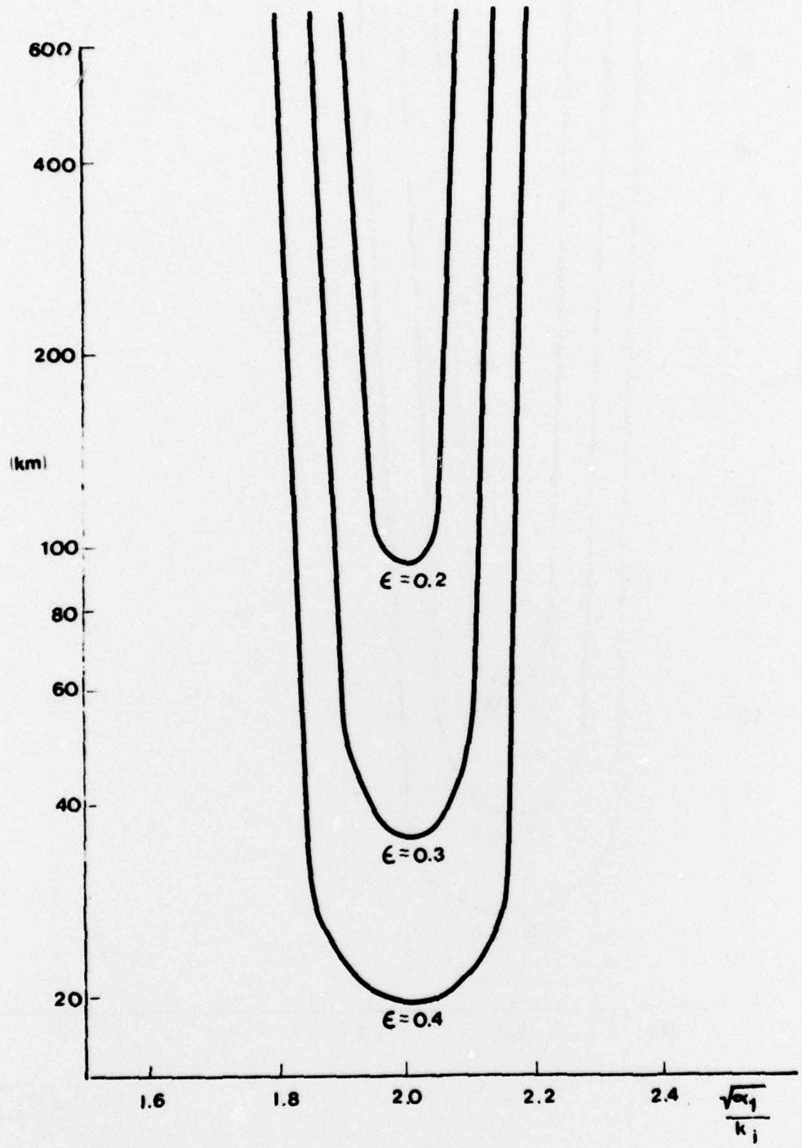

Fig. 5. Growth length for p = 2 mode with plasma parameters the same as in Fig. 3.

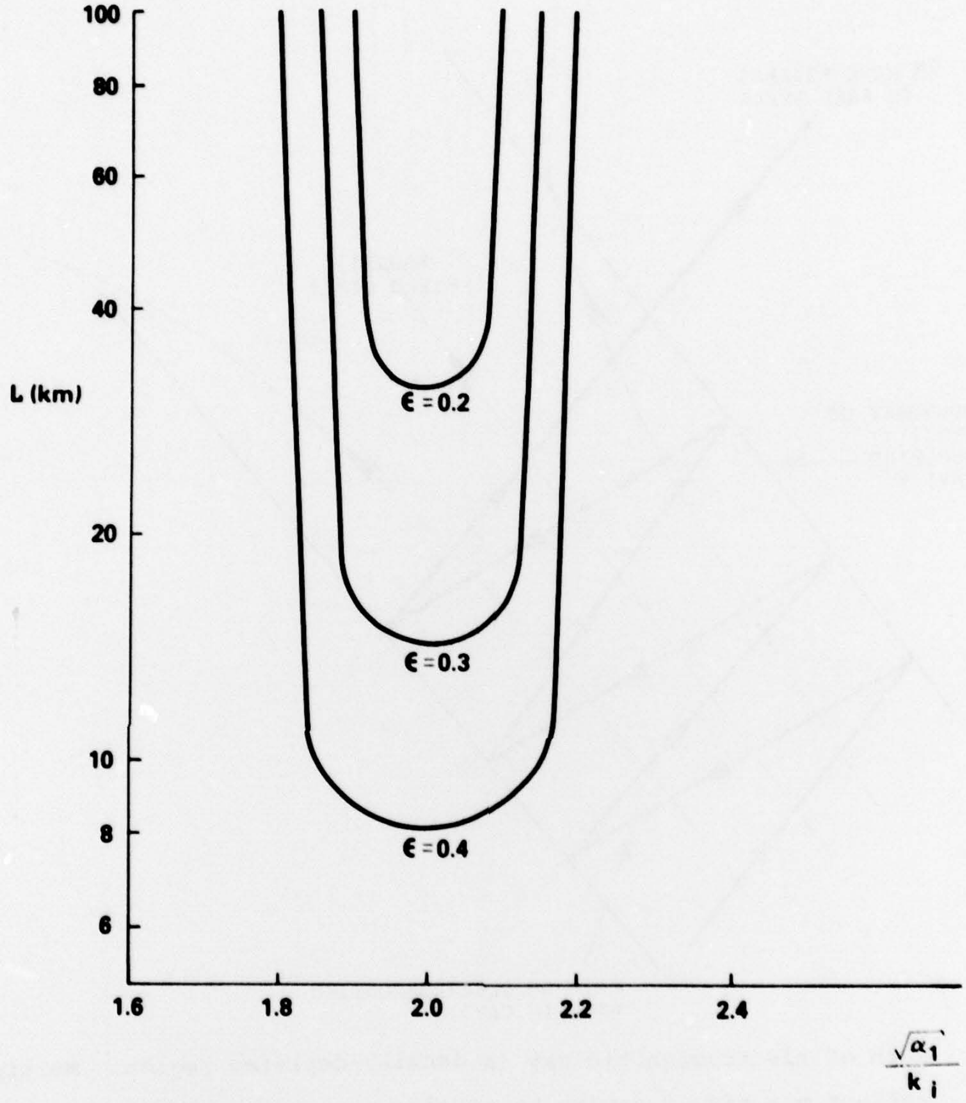

Fig. 6. Growth length for p = 2 mode with plasma parameters the same as in Fig. 4.

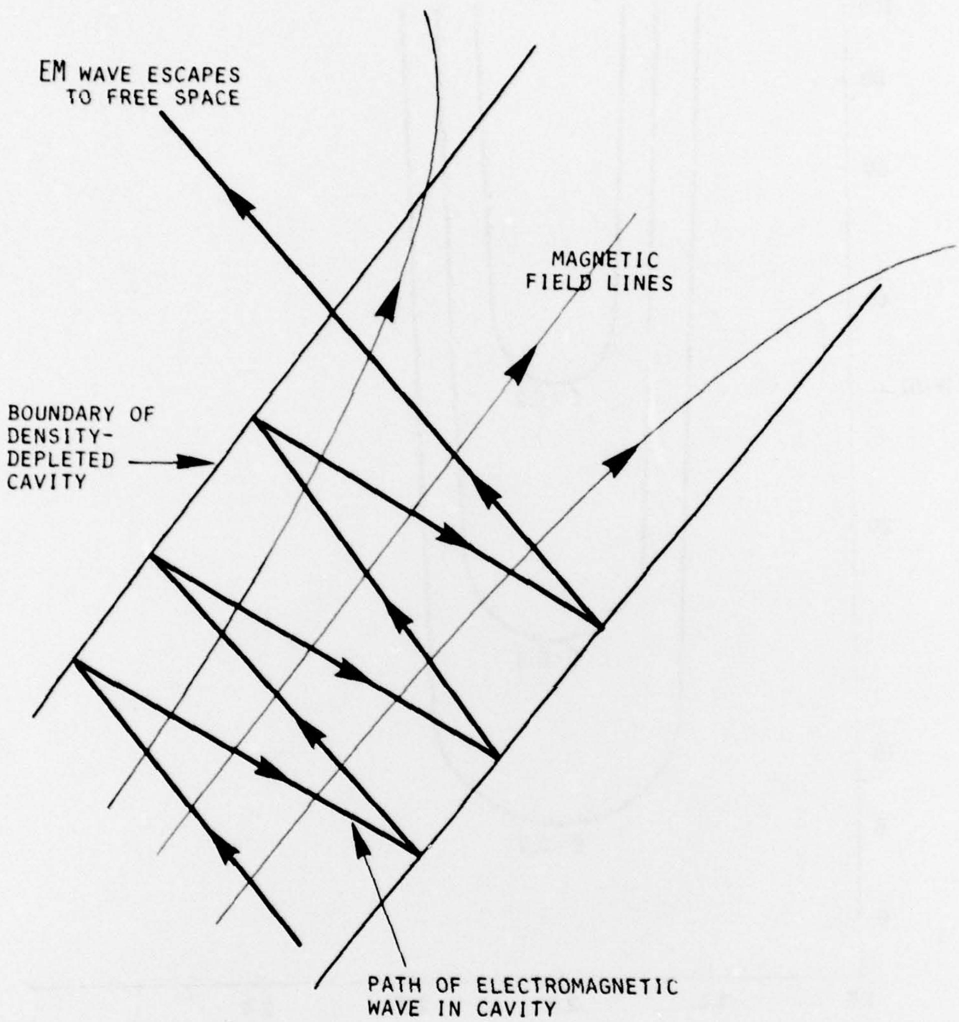

Fig. 7. Path of electromagnetic ray in density-depleted region. Multiple reflections off of cavity boundaries may provide long paths for growth. The way may eventually reach a weak field and/or low plasma density region where it is accessible to free space.

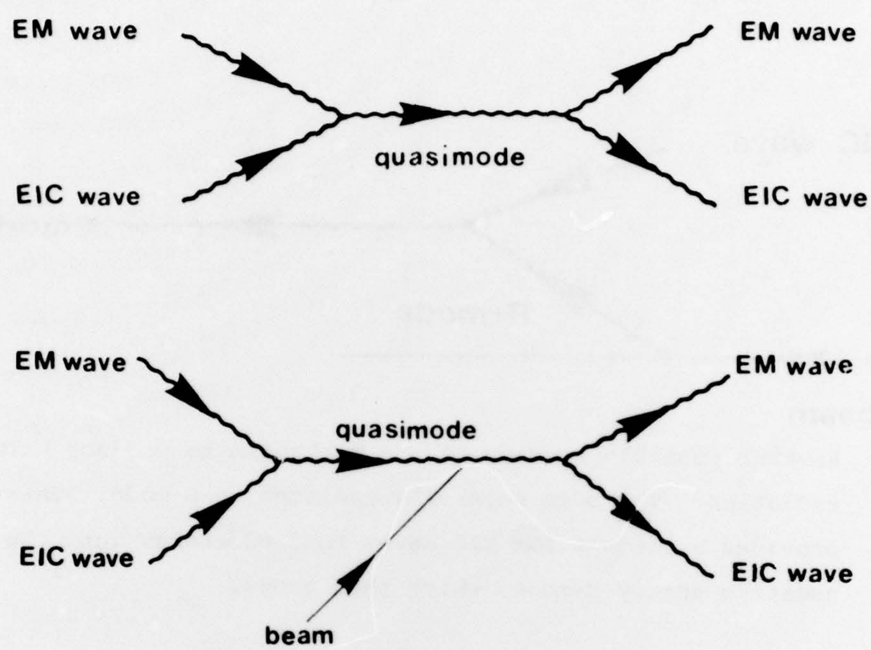

Fig. 8. (a) Simple resonant scattering of the EM and EIC waves. Energy transfer takes place in this scattering, normally from the EM wave to the EIC wave. (b) Resonant scattering of EM and EIC waves, in which the beat "quasimode" interacts with the beam at resonance. When the quasimode is negative energy in the beam frame, the beam transfers energy to it. When the electromagnetic wave is negative energy, energy flows from EIC to EM waves preferentially.

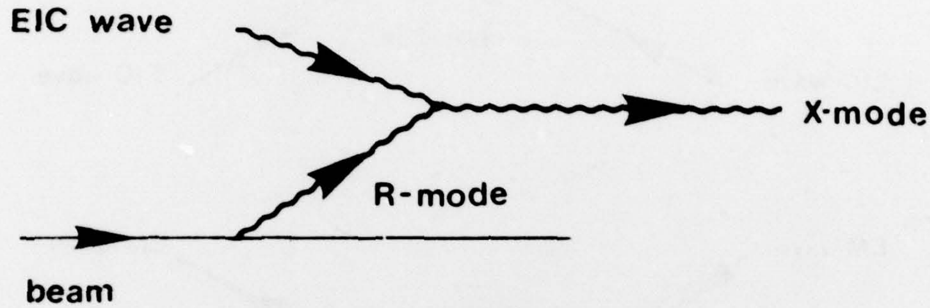

Fig. 9. Another possible version of the mechanism to produce kilometric radiation. The beam naturally radiates an R-mode. This mode provides a conduit for EIC waves to feed energy into the negative energy X-mode, which then grows.